5. Some other types of Lattice Design

(a) A lattice design in which the number of treatments is k^2, where k denotes block size (number of plots in a block)and the treatments are randomized with two restrictions as in L.S.D., is called a **lattice square deisng**. It means that a 2-restrictional and 2-dimensional lattice design is known as a lattice square design or quasi-latin square design.

(b) An experiment having $k(k+1)$ treatments can be conducted in incomplete blocks of size k with 1-restrictional randomization. Such a two-dimensional and one restrictional lattice design is known as **rectangular lattice** design. If $k = 2$ it is called **simple rectangular** lattice design and if $k = 3$ it is known as **triple** rectangular lattice design.

6. Layout of Balanced Lattice Design

We have already noticed that such a design is one restrictional and two dimensional in which the number of treatments is a perfect square (say k^2) and the size of incomplete blocks is its square root ($= k$).

We shall explain the method of layout for such a design with the help of an example.

Suppose that an experiment is conducted to compare 9 varieties of wheat as regards their yield in 3×3 balanced lattice design. Thus we have 9 treatments which may be represented as—1, 2, 3, 4, 5, 6, 7, 8 and 9. The treatments and their factorial correspondence can be shown as given below :

Treatments	:	1	2	3	4	5	6	7	8	9
Factorial combinations	:	00	01	02	10	11	12	20	21	22

Here the number of treatments is $k^2 = 9$

Number of plots per block is $k = 3$

Number of replications is $k + 1 = 4$

(to confound 4 treatments)

Note : The number of replications for k treatments shall be $(k+1)$ in order to fulfill the condition of balanced lattice design. The whole experimental area is divided into 4 replications and each replications is further divided into 3 incomplete blocks. Each incomplete block is divided into 3 plots. In order to allocate the treatments in first replication the 9 treatments are divided into 3 groups, say (1, 2, 3), (4, 5, 6), (7, 8, 9) and then these 3 groups of treatments are allocated randomly to the 3 blocks of the first replication. The 3 treatments of any group assigned to a block are allocated randomly to the 3 plots of this block.

The same procedure is adopted for the rest of the replication subject to the condition that every combination of 3 treatments shall appear once and only once in an incomplete block. The final layout of experiment can be as given below.

Replication I

Block	Treatments		
(1)	3	1	2
(2)	7	9	8
(3)	5	6	4

8

4

Replication II

Block	Treatments		
(4)	1	5	9
(5)	2	7	6
(6)	8	3	4

Rep-III

(7)	2	5	8
(8)	1	7	4
(9)	3	9	6

Rep-IV

(10)	1	8	6
(11)	2	4	9
(12)	3	5	7

Statistical analysis of a Balanced Lattice Design:

Step 1: First we calculate block totals (B), replication totals (R) for each block and each replication separately and finally the grand total (G).

Step 2: Now we calculate the treatment total (T) for each treatment taken over all the replication.

Step 3: Then we calculate for each treatment, the sum of block totals (B_t) over all the blocks in which that treatments t is present. Here $\Sigma B_t = kG$, k is block size and t denotes the serial number of treatments ($t = 1, 2, \ldots, 9$ in our example)

Step 4: Calculate $W = kT - (k+1) B_t + G$ for each treatment. Here $\Sigma W = 0$.

Step 5: Now we calculate the various sum of squares as follows.

$$C.F = \frac{G^2}{k^2(k+1)}$$

Total S.S or T.S.S $= \sum y^2 - C.F$

S.S. for replications $= \frac{\sum R^2}{k^2} - C.F$

Unadjusted S.S for treatments $= \frac{\sum T^2}{k+1} - C.F$

Adjusted S.S for blocks $= \frac{\sum W^2}{k^3(k+1)} - C.F$

S.S for Intra block error $=$ T.S.S $-$ S.S (replication)

$-$ unadjusted S.S (treatment)

$-$ adjusted S.S (blocks).

Step 6 : Now we prepare the ANOVA table as follows.

S.V	d.f	S.S	M.S.S	
Replication	k			
Treatment (unadjusted)	k^2-1			
Blocks (Adjusted)	k^2-1			
Intra block Error	$(k-1)(k^2-1)$	ES.S	E.M.S	
Total	$k^2(k+1)-1$			

Step 7: Then we calculate the value

$$\mu = \frac{\text{Block(adj) S.S} - \text{Intra block error (M.S)}}{k^2[\text{Block (adj) M.S}]}$$

If the value of μ comes out to be negative we take $\mu = 0$ and when $\mu = 0$ we skip the steps 8, 9 and 10.

If $\mu \neq 0$, then we follow steps 8, 9 and 10 also.

Step 8: We calculate the adjusted treatment total 'T'' for each treatment as

$$T' = T + \mu W.$$

and also calculate the adjusted treatment means 'M'' for each treatment as

$$M' = \frac{T'}{(k+1)}$$

Step 9: Then we calculate adjusted treatment mean square as

$$\text{Treatment M.S (adjusted)} = \left[\Sigma(T')^2 - \frac{G^2}{k^2}\right]\left[\frac{1}{(k+1)(k^2-1)}\right]$$

Step 10: After this, calculate effective error mean square as

$$\text{Effective error M.S} = (\text{Intra block EMS})(1 + k\mu).$$

Step 11 : Now Calculate

$$F = \frac{\text{Treatment M.S (adjus)}}{\text{Effective error M.S}} \quad ;(\text{if } \mu > 0)$$

$$F = \frac{\text{Treatment M.S (adjus)}}{\text{Intra block error M.S}} \quad ;(\text{if } \mu \leq 0)$$

Finally, the significance of value of F is tested.

Step 12 :

$$S.E(d) = \sqrt{\frac{2 \text{ Effective M.S}}{r}} \quad ; \text{if } \mu > 0$$

$$S.E(d) = \sqrt{\frac{2(\text{Intra block error M.S})}{r}} \quad ; \text{if } \mu \leq 0.$$

Then calculate

$$C.D = S.E(d) \times t \,(\text{for error d.f})$$